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BOUNDARY INTEGRAL EQUATION AND CONJUGATE GRADIENT METHODS FOR OPTIMAL BOUNDARY HEATING OF SOLIDS

R. A. MERIÇ
Marmara Scientific and Industrial Research Institute, Applied Mathematics Division, Gebze, Kocaeli, Turkey

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Abstract—A simple model problem in optimal boundary heating of solids is analyzed by numerical methods. The physical objective of the present "steady-state optimal control" problem is to achieve a desired temperature profile along a segment of the solid boundary with a minimum amount of a boundary heat flux which acts as the controlling function. The boundary integral equation methods are effectively used in the space discretizations of the necessary conditions for optimality of a performance index, which characterizes the physical goal mathematically. For minimization of the performance index the conjugate gradient method of optimization is utilized. Numerical results are presented for various values of the problem parameters which consist of a Biot number and a weighting parameter in the performance index. It is argued that the weighting parameter behaves like a free "design" parameter which controls the degree of achievement of the desired temperature profile versus the amount of power consumed through the boundary heating. It is also pointed out that the suggested numerical solution algorithm for the boundary control problem constitutes a new and efficient solution procedure which has distinct advantages over other available methods.

NOMENCLATURE

Bi,	Biot number;
$\mathbf{g}$,	gradient vector;
J,	performance index;
$\mathbf{s}$,	direction of search vector;
T,	temperature (state function);
$\mathbf{T}_0$,	temperature vector at $x = 0$;
u,	boundary heat flux (control function);
x, y,	Cartesian coordinates.

Greek symbols

α,	weighting parameter;
γ,	coefficient;
λ,	Lagrange multiplier (co-state function);
$\boldsymbol{\lambda}_1$,	co-state function vector at $x = 1$.

Subscript

m,	iteration number.

Superscript

T,	transpose of a matrix.

1. INTRODUCTION

THERE are many industrial processes in which it is required to control the temperature distribution in a given solid material. Optimal boundary heating constitutes by far the most common application of the controlled heating of solids. The mathematical problem falls into the general class of optimal control problems of distributed parameter systems (DPS). The state of the controlled system, for example, the temperature distribution in a solid, is usually governed by partial differential equations of various types [1]. When the state equation is elliptic and has no time variable involved, the problem becomes a steady-state control or static optimization problem [2]. If the state of the system is governed by a parabolic equation of the evolution type the control problem is then a dynamic one, and may be cast as an open-loop or feedback control problem [3–5].

The problem of determining the optimal control for DPS is generally very difficult to solve. Although it is possible to formulate the optimal control problem corresponding to many physical systems and to derive a set of optimality conditions, it is not easy to obtain the solution. Computational solutions appear mandatory for even the simplest of cases. In the past, numerical techniques have involved various space, space–time and time discretizations. Finite difference methods (FDM) have been utilized to produce simple computational algorithms which result in "acceptable" approximations to the optimal solutions [6]. Recently, finite element methods (FEM) have been applied to various steady and dynamic optimal control problems by the present author [2–5].

In this investigation, boundary integral equation methods (BIEM), or the boundary element methods (BEM), are applied to a steady-state optimal problem [7, 8]. Specifically, the problem constitutes an optimal boundary heating of a solid plate. The desired goal for the controlled heating is to bring the temperature of a part of the boundary to a desired level. Since the control function (in this case, the boundary heat flux) and also the desired "observation" are on the boundary of the solid only, the method of BIEM proposed for discretizing the system equations forms a very powerful numerical technique for such boundary control problems. Although the BIEM has successfully been applied to such diverse fields as elastostatics and elastodynamics, plasticity, thermoelasticity and thermoplasticity, heat conduction and free surface flows, the method has not been utilized in optimal control problems of DPS so far in the literature [7, 9–13]. Besides the space discretizations of the system equations by the BIEM, a given performance index which characterizes the desired physical goal is minimized by the conjugate gradient method of mathematical programming [15].

2. STATEMENT OF THE PROBLEM

A simple model problem in optimal control of steady-state heat transfer will be analyzed in a square solid plate. In particular, it is desired to bring the temperature of a part of the boundary of the plate to a certain level through an optimal boundary heat flux acting as the control function. The 2-dim. differential equation which characterizes the steady-state heat conduction in the homogeneous plate may be written in a nondimensional form as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \quad 0 \le x, y \le 1 \tag{1}$$

where $T = T(x, y)$ represents the state of the system, i.e. the temperature distribution in the plate. It is assumed that there are no distributed heat sources in the solid acting as distributed control functions (cf. [2, 3]). The elliptic partial differential equation (1) requires a boundary condition to be specified at every point of the boundary and they are prescribed as

$$x = 0; \quad \frac{\partial T}{\partial x} = 0, \tag{2}$$

$$x = 1; \quad \frac{\partial T}{\partial x} = u(y), \tag{3}$$

$$y = 0; \quad T = 1, \tag{4}$$

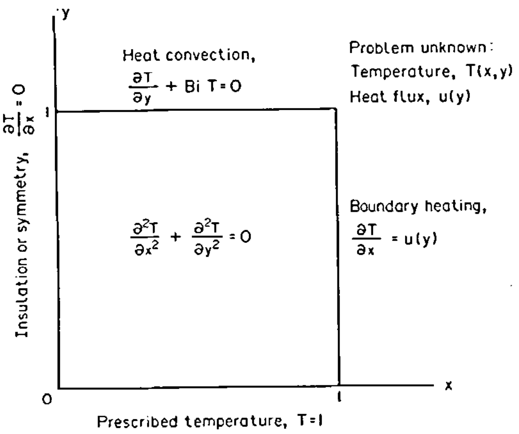

FIG. 1. Problem geometry and system equations.

$$y = 1; \quad \frac{\partial T}{\partial y} + Bi\, T = 0 \tag{5}$$

where Bi is the dimensionless Biot number indicating the ratio of the surface conductance to the conduction of solid, and $u(y)$ is the unknown boundary heat flux representing the control function. As such, the problem is a typical example of a boundary control problem [4, 5]. The homogeneous third kind of boundary condition (5) may behave as the second or first kind as the given Biot number Bi tends to zero or infinity. The problem geometry and the system equations are shown schematically in Fig. 1.

If the function $u(y)$ at $x = 1$ were given explicitly the heat transfer problem defined by equations (1)–(5) would have been easy to solve even analytically depending on the behavior of u. However, the function $u(y)$ is not given *a priori* in the problem but constitutes an unknown control function to be determined, as well as the state function $T(x, y)$. The control function will be so chosen that a given physical objective is satisfied.

Specifically this objective is to bring the temperature of the part of the boundary at $x = 0$ to a level of 1 in an "average" sense. Thus, a suitable measure of the nearness of the boundary temperature to that desired will have to be chosen. The present control problem deviates from the type of control problem known as the "exact" control problems in which the boundary temperature at $x = 0$ would have been required to be 1 "exactly" at every point along the boundary segment.

The problem would not be well-posed without some form of constraint on the control function u, i.e. the boundary heat flux at $x = 1$. This constraint may be taken as forcing the heat flux as near as zero, again in an "average" sense. Thus, the stated objectives of the problem may be cast into a mathematical form as defined by a performance index J

$$J = \frac{1}{2} \int_0^1 \{[T(0, y) - 1]^2 - \alpha u^2(y)\}\, dy \tag{6}$$

where α is a given weighting parameter. The first term in the above quadratic functional is the integral of the square of the deviation of the temperature from the desired temperature level 1 over the boundary section at $x = 0$. The second term is, on the other hand, the integration of the square of the control function $u(y)$ with a weighting coefficient over the section of the boundary where $x = 1$.

The problem parameter α plays an important role in the problem. It combines actually two physical objectives in a linear combination by weighting. The previously stated physical objectives of the problem are attained with a relative degree of achievement according to the value of α when the performance index J is minimized. It can be argued that taking a smaller value for α would result in the boundary temperature at $x = 0$ nearer to the desired level. Nevertheless, if the fuel cost necessary for the boundary heat flux is relatively mportant one might not choose a very small value for α. Hence, the function $u(y)$ which minimizes J under :he system constraints (1)–(5) is the desired optimal ıeat flux solution.

3. METHOD OF SOLUTION

3.1. *Necessary conditions for optimality*

The steady-state boundary control problem investigation may be reformulated by finding the necessary conditions for optimality, i.e. the necessary conditions for J to be an extremum while the heat conduction equation (1) and the boundary conditions (2)–(5) are satisfied. These conditions which are a set of partial differential equations and some transversality conditions prescribed on the boundary of the domain can be derived by applying Pontryagin's minimum principle [6] (basically a calculus of variations method) to the optimal control problem in question. Thus, combining the performance index J with the state equation constraint (1) by means of a Lagrange multiplier (also called adjoint or co-state function) $\lambda(x, y)$, and then forcing the first variation of J equal to zero under the given boundary constraints (2)–(5) render the following conditions for optimality:

State equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0. \tag{7}$$

Co-state equation.

$$\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} = 0. \tag{8}$$

Transversality conditions.

$$x = 0; \quad \frac{\partial T}{\partial x} = 0, \tag{9}$$

$$x = 0; \quad \frac{\partial \lambda}{\partial x} + T = 1, \tag{10}$$

$$x = 1; \quad \frac{\partial T}{\partial x} = u, \tag{11}$$

$$x = 1; \quad \frac{\partial \lambda}{\partial x} = 0, \tag{12}$$

$$x = 1; \quad \lambda + \alpha u = 0, \tag{13}$$

$$y = 0; \quad T = 1, \tag{14}$$

$$y = 0; \quad \lambda = 0, \tag{15}$$

$$y = 1; \quad \frac{\partial T}{\partial y} + Bi\, T = 0, \tag{16}$$

$$y = 1; \quad \frac{\partial \lambda}{\partial y} + Bi\, \lambda = 0. \tag{17}$$

We may note that the linear equations (7)–(17) involve three unknown functions, namely, $T(x, y)$, $\lambda(x, y)$ and $u(y)$. Since the "observation" and also the control is on the boundary of the solution domain, the state and co-state equations are not coupled directly, as would have been the case in distributed "observation" and/or control cases [2–5]. The state function T, co-state function λ and control function u are coupled through the transversality conditions prescribed on the boundary. Equation (13) may also be shown to be the gradient of the performance index J with respect to the control function u [6].

It is possible to eliminate u from the set of optimality conditions through equations (11) and (13), thus leaving the conditions in terms of T and λ only. Hence, the present steady-state optimal control problem may be formulated as a boundary value problem of mathematical physics. Since further combining of T and λ appears to be difficult any numerical solution of the problem will involve an iterational procedure between the two unknown functions. In the present method of solution, instead of treating the problem as a usual boundary value problem an optimizational technique will be adopted in which the performance index J is minimized by the conjugate gradient method of mathematical programming.

3.2. *Treatment of the optimal control problem as a mathematical programming problem*

The steady-state optimal control problem requires the minimization of the performance index J subject to the state and co-state partial differential equations and the relevant transversality conditions. The stated problem may be formulated as a standard mathematical programming problem if the infinitely large number of variables inherent in the continuous-space optimal control problem are reduced to a finite number through any kind of space discretization, for example, finite differences, finite elements or boundary elements. In the present study the BIEM is chosen to discretize the continuous space variables since the method offers some special advantages for this specific boundary control problem over other available "domain" type methods.

3.3. *Boundary element discretizations of the state and co-state equations*

The BIEM (or BEM) is a powerful means of solving field problems in continuum mechanics. In this method the field equations are transformed into a set of integral equations on the boundary by a weighted residual formulation in which the fundamental solution of the original problem is utilised as the weighting function. Since one than deals with an integral equation defined along the boundary, the dimensionality of the problem is practically reduced by one. Although the solution to the integral equation only provides values along the boundary of the domain, solutions for interior points may be found by numerical quadratures, if desired.

In this study, the BIEM is adopted to discretize the state and co-state equations (7) and (8), alternatively. The set of optimality conditions and the performance index J are such that there is even no need to evaluate the solution functions in the interior of the domain.

Let us call the state equation (7) and the boundary conditions (9), (11), (14) and (16) as the T-problem. If we assume for a moment that the control function $u(y)$ is known explicitly, then the T-problem could be solved, for example, by the BIEM. The solution of a Laplace equation with mixed boundary conditions by the BIEM is analyzed in ref. [7]. In fact, it contains a simple computer program in FORTRAN for the numerical solution of such potential problems. Although only the first and second kind of boundary conditions are treated in the program, the incorporation of the third kind of the boundary condition, equation (16), may be done very easily [8].

Similarly, the co-state equation (8) and the boundary conditions (10), (12), (15) and (17) will be called the λ-problem which constitutes another potential problem. Solution of this potential problem by the BIEM may proceed easily if the temperature values at $x = 0$ were known explicitly.

An iterational method of solution will be adopted to find the optimal control function u. First of all, the boundary of the solution domain is divided into elements [7]. Initial numerical values are then assumed for the control vector $\mathbf{u}_0$ at the interconnecting points of the boundary elements of $x = 1$. The subscript 0 indicates that the vector is an initial guess vector. On the other hand, linear boundary elements are chosen for the T- and λ-problem, that is, both T and λ functions and their normal derivatives are interpolated linearly over each boundary element [7].

Using the $\mathbf{u}_0$ vector for the boundary condition (11), solution of the T-problem then proceeds according to the standard BIEM resulting in the solution of the nodal values of the temperature T and its normal derivative along the entire boundary. In particular we may call the resultant temperature nodal values at $x = 0$ by the vector $(\mathbf{T}_0)_0$ where the inner subscript 0 refers to the temperature values at $x = 0$, while the outer subscript 0 indicates that the solution corresponds to the initial control vector $\mathbf{u}_0$.

The λ-problem may be solved similarly by the BIEM using the previously obtained $(\mathbf{T}_0)_0$ vector in the boundary condition (10). Only the λ values along the boundary section $x = 1$ are needed to calculate the gradient of J by equation (13). Thus, these λ values may be denoted by the vector $(\lambda_1)_0$ where the subscript 1 refers to the boundary segment at $x = 1$ and the subscript 0 indicates that the vector corresponds to the $(\mathbf{T}_0)_0$ and consequently to the $\mathbf{u}_0$ vectors.

It is now possible to evaluate the performance index J, equation (6), and its gradient with respect to the control function u, equation (13), by utilizing the solution vectors $(\mathbf{T}_0)_0$ and $(\lambda_1)_0$, and also the initial guess vector $\mathbf{u}_0$. Better estimates of the control vector are to be found by the conjugate gradient method of optimization which is considered next.

3.4. *Conjugate gradient method*

The steady-state optimal control problem may now be solved by the conjugate gradient method of mathematical programming after the problem has been discretized in space by use of the BIEM. Standard FORTRAN programs exist in the literature for the optimization method which finds the unconstrained minimum of a multivariable, nonlinear function [14]. The basic procedure of the conjugate gradient method is described by Fletcher and Reeves [15]. The method is an iterative unconstrained optimization technique based on the calculation of the gradient of the function to be minimized. It has been applied to optimal control problems with good results previously in the literature [5].

In the present problem the function to be minimized is the performance index J taken as a nonlinear function of the discretized control vector $\mathbf{u}$. Hence, at the mth iterational level the gradient of J with respect to the vector $\mathbf{u}_m$ can be found by means of equation (13) and is given as the gradient vector $\mathbf{g}_m$ that is

$$\frac{\partial J}{\partial \mathbf{u}_m} = \mathbf{g}_m = (\lambda_1)_m + \alpha \mathbf{u}_m. \tag{18}$$

The complete numerical method of solution for the optimal control problem, including the iterational method of conjugate gradient technique, can be summarized as follows:

1. For $m = 0$, guess the initial control vector $\mathbf{u}_m$.
2. Solve the T-problem using $\mathbf{u}_m$ to obtain $(\mathbf{T}_0)_m$ vector.
3. Solve the λ-problem using $(\mathbf{T}_0)_m$ to obtain $(\lambda_1)_m$ vector.
4. Calculate the gradient vector $\mathbf{g}_m$ by equation (18).
5. Calculate the conjugate gradient parameter μ_m given by

$$\mu_m = \frac{\mathbf{g}_m^T \mathbf{g}_m}{\mathbf{g}_{m-1}^T \mathbf{g}_{m-1}}, \quad \text{with } \mu_0 = 0 \text{ if } m = 0$$

where the superscript T refers to the transpose of a vector.

6. Calculate the direction of search vector $\mathbf{s}_m$ where

$$\mathbf{s}_m = -\mathbf{g}_m + \mu_m \mathbf{s}_{m-1}.$$

7. Iterate on the control

$$\mathbf{u}_{m+1} = \mathbf{u}_m + \gamma_m \mathbf{s}_m$$

where γ_m is the coefficient determined by performing a 1-dim. minimization along the direction of search.

8. Increase m and repeat steps 2–7 until specified convergence criteria are satisfied.

During the above minimization procedure the evaluation of J is needed for each iteration level m. Such evaluations are done by approximating the definite integrals in J in terms of finite sums of integrands with weighting coefficients, that is by Gauss quadratures.

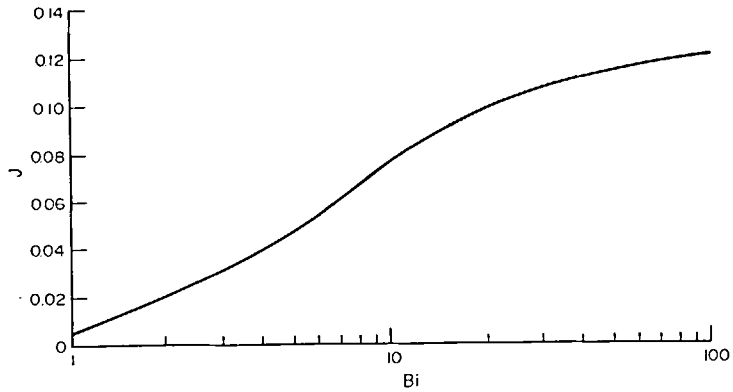

FIG. 2. Performance index J for $\alpha = 0.001$.

4. NUMERICAL RESULTS

Numerical results are presented for the model problem of optimal boundary heating of a square plate. The given control problem involves two parameters, namely Bi and α. It is interesting to note the effects of limiting values of these parameters on the optimal solutions. When the dimensionless Biot number Bi is identically zero, the boundary condition (5) at $y = 1$ reduces to the insulation boundary condition. In this case the optimal solution of the control problem is a trivial one for any value of α. That is, no boundary heating would be necessary with $u = 0$ as the temperature distribution in the plate will be $T(x, y) = 1$ for $Bi = 0$. For this case, the quadratic performance index J attains its absolute minimum, i.e. J becomes equal to zero.

If the Biot number Bi tends to infinity the boundary condition (5) reduces to the prescribed temperature boundary condition $T = 0$ at $y = 1$. This limiting value of Bi results in the requirement of the greatest amount of boundary heating at $x = 1$ to obtain the physical objective as nearly as possible for a fixed value of α.

As the weighting parameter α tends to zero there is less and less constraint on the control function u. Thus, the boundary control function may be chosen "freely" to meet the desired objective. At the other extreme with α having a very large value the temperature distribution at the boundary section $x = 0$ would hardly reach the desired level of 1 (for large values of Bi) due to the heavy constraint that forces the control function to zero. In fact, if both α and Bi are taken as infinitely large the temperature distribution in the whole plate would be a linear function ranging from $T = 1$ at $y = 0$ to $T = 0$ at $y = 1$.

In Fig. 2, the performance index J is plotted against the Biot number Bi for $\alpha = 0.001$. It may be noticed from the figure that for a fixed degree of constraint on u (i.e. for a fixed value of α) the dependency of J is almost asymptotic for small and large values of Bi.

In Fig. 3, the optimal boundary heat flux u is plotted as a function of y for different values of the weighting parameter α when Bi is fixed as 5. For smaller values of

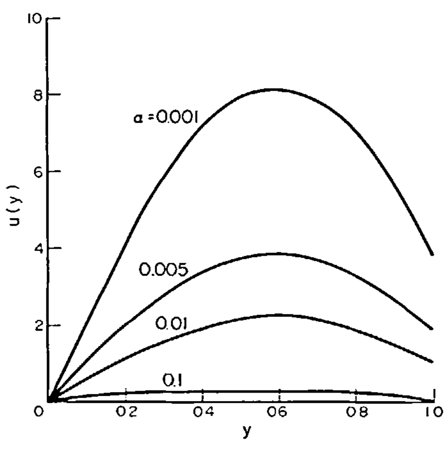

FIG. 3. Optimal control function $u(y)$ for different α values when $Bi = 5$.

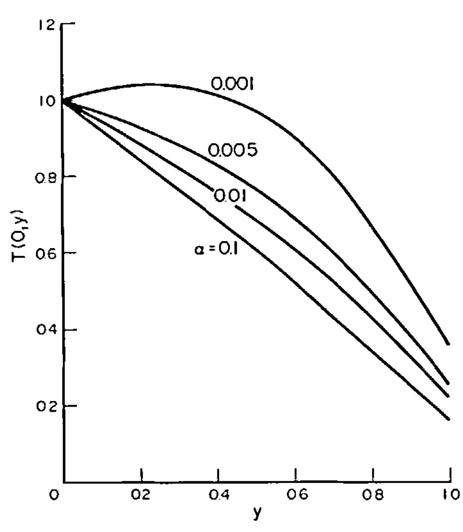

FIG. 4. Boundary temperature $T(0, y)$ for different α values when $Bi = 5$.

α, the control function u adopts higher distributed values as shown in the figure. Figure 4 represents the boundary temperature distribution at $x = 0$ against the space coordinate y for different α values when $Bi = 5$. Thus the last two figures are the optimal solutions corresponding to the same set of α values when the Biot number Bi is held fixed.

Figures 5 and 6 represent the optimal solutions when the weighting parameter α is held fixed at 0.001. In Fig. 5 the optimal control function u is shown as a function of y for different values of Bi, and correspondingly the boundary temperature at $x = 0$ is shown for the same set of Bi values in Fig. 6.

5. CONCLUSIONS

A simple model problem in optimal control heat transfer is analyzed by numerical methods. In particular the heating of a square solid plate by an optimal boundary heat flux is investigated. The physical objectives of the problem are taken as to bring the temperature of a part of the boundary to a desired level by applying a "minimum" amount of a boundary heat flux.

First, the necessary conditions for optimality of a performance index which characterizes the physical objectives are found by calculus of variations using a Lagrange multiplier technique. These necessary conditions are then discretized in space by using the boundary integral equation methods. Minimization of the performance index by the conjugate gradient method of optimization then yields the optimal solutions of the problem, that is, the optimal boundary heat flux and the corresponding temperature distribution in the plate. Some conclusions may be drawn from the analysis of the control problem, which may be given as follows:

(1) The present model problem involves two parameters which strongly influence the optimal solutions. The Biot number Bi is a physical parameter which may be fixed in a given situation. The weighting parameter α may, on the other hand, be interpreted as a free "design" parameter, which has to be chosen and adjusted in the light of experience and computer results to achieve the stated objectives on a relative basis. The cost of the fuel consumption for the boundary heat flux may play an important role in choosing the value for α.

(2) As the control problem falls into the class of boundary control, the boundary integral equation methods constitute especially very efficient numerical techniques of space discretization, with no need of any domain integrations.

(3) Although the finite element or finite difference methods could have been utilized for the present model problem, the boundary integral equation methods have a distinct advantage over such domain type of methods in that the dimensionality of the problem is practically reduced by one since one deals with integral equations defined over the boundary only.

(4) The "observation" and "control" may be present in the domain as well as on the boundary in other control problems. Such cases would arise if, for example, the temperature profile of the whole plate is controlled by a distributed heat source in the plate. The boundary integral equation methods could still be used for such problems. However, simple numerical "domain" quadratures would be necessary in order to evaluate some "source" terms in the partial differential equations of the necessary conditions for optimality.

(5) Computer programs available in the literature for the boundary integral equation and conjugate gradient methods are very helpful in the application of these methods to a wide variety of optimal control problems in heat transfer.

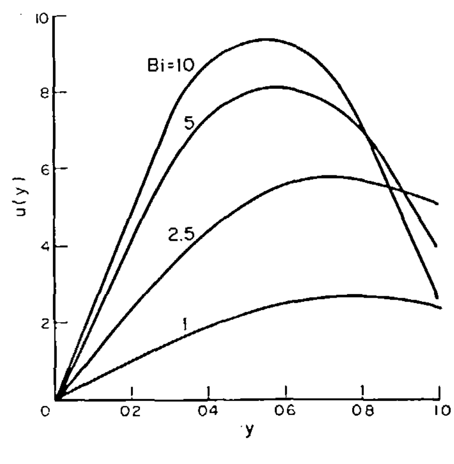

Fig. 5. Optimal control function $u(y)$ for different Bi values when $\alpha = 0.001$.

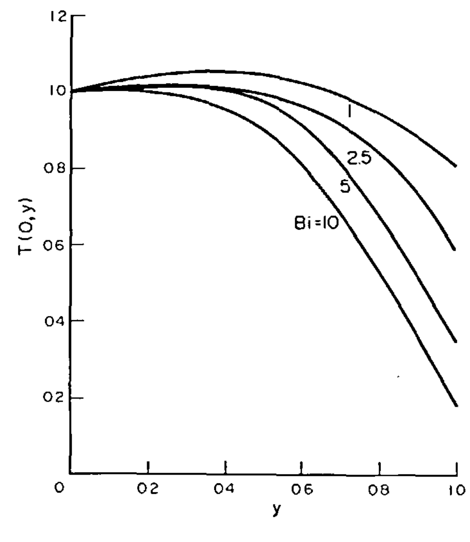

Fig. 6. Boundary temperature $T(0, y)$ for different Bi values when $\alpha = 0.001$.

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METHODES DE L'EQUATION INTEGRALE LIMITE ET DU GRADIENT CONJUGE POUR LE CHAUFFAGE OPTIMAL DES SOLIDES AUX FRONTIERES

Résumé—Un modèle simple de problème de chauffage optimisé de solides par les frontières est analysé par des méthodes numériques. Le but physique de ce problème à "commande optimale permanente" est d'obtenir un profil de température donné le long d'un segment de la frontière solide avec une quantité de chaleur minimale. Les méthodes de l'équation intégrale limite sont utilisées dans les discrétisations spatiales des conditions nécessaires pour l'optimalité d'un indice de performance qui caractérise mathématiquement l'objectif physique. On utilise la méthode du gradient conjugué pour minimiser l'indice de performance.

Des résultats numériques sont présentés pour différentes valeurs des paramètres du problème qui sont un nombre de Biot et un paramètre de pondération dans l'indice de performance. Ce dernier paramètre agit comme un paramètre libre "de conception" qui commande le degré d'achèvement du profil de température désiré en fonction de la quantité d'énergie consommée à travers la paroi chauffée. On montre aussi que la solution numérique algorithmique constitue une nouvelle procédure efficace qui a des avantages distincts de ceux des autres méthodes connues.

GRENZ-INTEGRAL-GLEICHUNG UND KONJUGIERTE GRADIENTEN VERFAHREN FÜR OPTIMALE HEIZUNG EINER FESTSTOFFBERANDUNG

Zusammenfassung—Ein einfaches Modellproblem zur optimalen Randbeheizung von Festoffen wird mit numerischen Verfahren untersucht. Das physikalische Ziel dieses "steady-state optimal control". Problems ist es, ein vorgegebenes Temperaturprofil entlang eines Segments der Feststoffberandung mit minimalem Gesamtwärmestrom, der als Kontrollfunktion dient, zu erhalten. Die Grenz-Integral-Gleichungs-Verfahren wurden vorwiegend zur Optimierung eines Leistungsindex, der das physikalische Ziel mathematisch charakterisiert, bei der Diskretisierung der geforderten Bedingungen benutzt. Zur Minimierung des Leistungsindex wurde das konjugierte Gradientenoptimierungsverfahren verwendet.

Numerische Ergebnisse werden für verschiedene Werte des Problemparameters, der aus einer Biot-Zahl und einem Wichtungs-Parameter im Leistungsindex besteht, gezeigt. Es wird bewiesen, daß sich der Wichtungsparameter wie ein freier "Form"-Parameter verhält, der den Grad der Übereinstimmung eines bestimmten Temperaturprofils in Abhängigkeit von der durch die Randheizung verbrauchten Gesamtleistung kontrolliert. Es wird zusätzlich hervorgehoben, daß der vorgeschlagene numerische Lösungsalgorithmus für Probleme der Randkontrolle einen neuen und wirkungsvollen Lösungsweg darstellt, der verschiedene Vorzüge gegenüber anderen verfügbaren Methoden besitzt.

ИСПОЛЬЗОВАНИЕ ГРАНИЧНОГО ИНТЕГРАЛЬНОГО УРАВНЕНИЯ И СОПРЯЖЕННЫХ ГРАДИЕНТНЫХ МЕТОДОВ ДЛЯ ОПРЕДЕЛЕНИЯ ОПТИМАЛЬНОГО ГРАНИЧНОГО НАГРЕВА ТВЕРДЫХ ТЕЛ

Аннотация—Численными методами анализируется простая модель оптимального граничного нагрева твердых тел. С физической точки зрения проблема "стационарного оптимального контроля" заключается в достижении необходимого профиля температур вдоль участка твердой границы при минимальной величине теплового потока на границе, играющего роль контролирующей функции. Методы граничного интегрального уравнения эффективно используются для пространственной дискретизации условий, необходимых для получения оптимального режимного индекса, математически характеризующего физическую цель задачи. Для минимизации режимного индекса используется сопряженный градиентный метод оптимизации.

Представлены численные значения различных параметров задачи, входящих в режимный индекс, включающий число Био и взвешенный параметр. Высказано сомнение в справедливости утверждения, что взвешенный параметр ведет себя как свободный "расчетный" параметр, определяющий степень достижения необходимого профиля температур в зависимости от количества поглощаемой за счет граничного нагрева энергии. Показано также, что предложенный алгоритм численного решения задачи граничного контроля представляет собой новый и эффективный метод решения, который обладает явными преимуществами перед другими имеющимися методами.